Axle Calculations

This document describes the axle calculations for the shredder. It is subdivided in the following manner:

1. Linear deflection and stresses.
2. Angular deflection and stresses.

The following characteristics are known, where the axle is a hollow square tube of 50x50x3 mm:

Code	Description	Value	Unit
Material	S235 JRH (steel)	-	-
E_{axle}	Young's Modulus of axle	205 - 215	GPa
G_{axle}	Shear Modulus of axle	78.9 - 82.7	GPa
τ_{axle}	Shear strength of axle	136 - 158	MPa
a	Outer width axle	50	mm
b	Inner width axle	44	mm
σ_{ys}	Yield strength of axle	235 - 274	MPa

Table 1 *Axle characteristics (E, G, shear and yield values from Granta EduPack)*

1.1 Linear Deflection

The axle is expected to deform under the static load of the blades and filler rings. These are both seen on the Instructables page. The axle is made from S235 JRH steel. The blades and filler rings result in a distributed load on the axle. The overhang between the supported sides (bearings) is determined by the number of blades and spacers in equation (1.1).

$$L = n_{blades} \cdot t_{blades} + n_{spacer} \cdot t_{spacer} \tag{1.1}$$

$$L = 24 \cdot 0.015 + 13 \cdot 0.006 = 0.438\, m \tag{1.1r}$$

Where n represents the number of blades and spacers, and t the thickness in meters. For the current shredder this gives the following value of 0.438 m.

The distributed load given by the blades and spacers is determined using (1.2).

$$q_{blades} = \frac{F}{t} = \frac{m \cdot g}{t} = \frac{3.021 \cdot 9.81}{0.015} = 1975.734 \text{ N/m} \tag{1.2a,r}$$

$$q_{spacer} = \frac{F}{t} = \frac{m \cdot g}{t} = \frac{0.092 \cdot 9.81}{0.006} = 150.42 \text{ N/m} \tag{1.2b,r}$$

$$q_{axle} = \frac{m \cdot g}{L} = \frac{1.87464 \cdot 9.81}{0.438} = 41.9868 \text{ N/m} \tag{1.2c,r}$$

Where q is the distributed load in N/m, F the force in N, t the thickness in m, m the mass in kg, g the acceleration due to gravity in m/s^2. The mass is specific for each component respectively.

The average of the distributed load is taken to simplify the equation. This is done by first determining the total load on the axle, and then dividing this number by the length of the axle. The average is determined in 1.2d and this represents q_{ave}, once again in N/m.

$$q_{ave} = \frac{g \cdot (n_{blades} \cdot m_{blades} + n_{spacer} \cdot m_{spacer} + 1.87454)}{L} \tag{1.2d}$$

$$q_{ave} = \frac{9.81 \cdot (24 \cdot 3.021 + 13 \cdot 0.092 + 1.87454)}{0.438} \cong 1692.663 \text{ N/m} \tag{1.2d,r}$$

The maximum bending angle and deflection due to a distributed load is determined via (1.3) and (1.4).

$$\varphi_{max} = \frac{q \cdot L^3}{6 \cdot E \cdot I} \tag{1.3}$$

$$f_{max} = \frac{q \cdot L^4}{8 \cdot E \cdot I} \tag{1.4}$$

Where φ is the angle in radians and f the deflection in meters. The E is the Modulus of Elasticity and is taken as a constant for the material. The I_z for a hollow tube is determined using (1.5). Where the outer is the a-value and the inner the b-value, see table 1.

$$I_z = \frac{1}{12} w_o \cdot h_o^3 - \frac{1}{12} w_i \cdot h_i^3 \tag{1.5}$$

$$I_z = \frac{1}{12}(0.05) \cdot 0.05^3 - \frac{1}{12}(0.044) \cdot 0.044^3 \cong 2.085 \cdot 10^{-7}\, m^4 \tag{1.5,r}$$

The beam is taken for half the length to simplify the calculations. Now the beam is taken as being clamped in the middle, as the angle is at zero for the maximum deflection point. Thus the length is taken as half of the total length. The value of the Young's Modulus is taken from Granta EduPack: the range is 205-215 GPa. For the calculation maximum deflection is discussed, thus the lowest E is taken.

$$f_{max} = \frac{q \cdot L^4}{8 \cdot E \cdot I} = \frac{1692.663 \cdot 0.219^4}{8 \cdot 205 \cdot 10^9 \cdot 2.085 \cdot 10^{-7}} = 0.11 \cdot 10^{-5} \text{ m} \tag{1.6,r}$$

The maximum deflection in the middle of the axle is thus only 0.01 mm, this result is an estimation as the average of the balanced force was taken.

1.2 Linear Stresses

The full axle length gives the V-diagram and M-diagram. These give the peak of the moment and shear force. With these the internal stresses are determined. Both the bending as the shear stress is calculated.

$$\sigma_{max} = \frac{M \cdot c}{I} = \frac{81.18 \cdot 0.025}{2.085 \cdot 10^{-7}} = 9.7\ MPa \qquad (1.7,r)$$

The maximum bending stress may not exceed the yield stress of this material, taken as 235 MPa from Granta EduPack. The axle will not fail due to the bending stress of 9.7 MPa.

The shear stress in the axle at a certain point is calculated in the following manner. The formula states that the maximum shear is found for the largest first moment area [Q] and the smallest thickness [t]. In the axle this is found in the middle. Thus, it is assumed that the shear stress is at its maximum in the middle.

$$\tau_{max} = \frac{V \cdot Q}{I \cdot t} \qquad (1.8)$$

$$Q = \bar{y}' \cdot A' \qquad (1.9)$$

$$\bar{y}' = \frac{\sum \bar{y}_i A_i}{\sum A_i} \qquad (1.10)$$

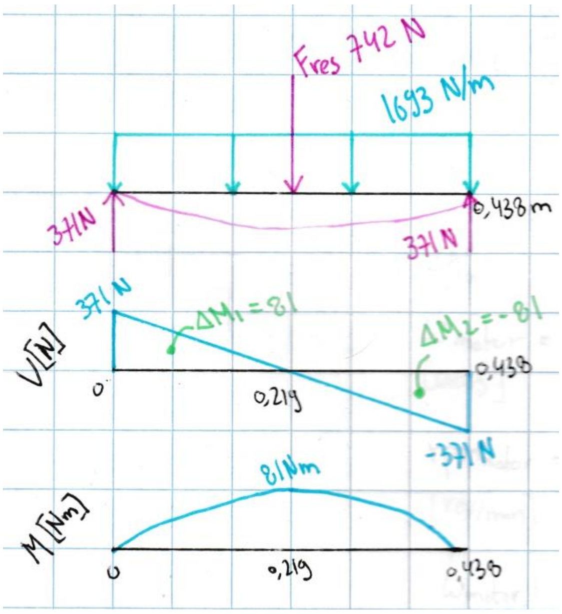

Figure 1 *Reaction forces, V-diagram and M-diagram*

Equation (1.10) gives the median for the sheared area used for the calculation of the first moment of area in (1.9).

$$\bar{y}' = \frac{\sum \bar{y}_i A_i}{\sum A_i} = \frac{(75 \cdot 12.5) + (75 \cdot 12.5) + (132 \cdot 23.5)}{75 + 75 + 132} \cong 17.65\ mm \qquad (1.10,r)$$

$$Q = \bar{y}' \cdot A' = 17.65 \cdot (75 + 75 + 132) = 4977\ mm^3 \qquad (1.9,r)$$

$$\tau_{max} = \frac{V \cdot Q}{I \cdot t} = \frac{371 \cdot 4977 \cdot 10^{-9}}{2.085 \cdot 10^{-7} \cdot 0.006} = 1.47\ MPa \qquad (1.8,r)$$

The shear stress is smaller than the shear strength of 136 MPa of the axle. Thus, the axle will not fail due to the shear stresses.

2.1 Angular Deflection

The axle is expected to deform with a torsion angle due to the torque applied during the shredding process. The torsion angle is represented in a table over a range of torque, as the applied torque will vary with future power improvements. The torque range is derived from the equations seen in chapter 1c. The following equation describes the angle of twist due to an applied torque over a set length of the axle. The angle twist Φ is in radians, the torque T in Nm, G the shear modulus in N/m^2 and J the second polar moment in m^4.

$$\Phi = \frac{T \cdot L}{G \cdot J} \tag{2.1}$$

The second polar moment for a hollow tubular shaft is determined via (2.1). The used shaft has rounded corners. This shape is simplified to right angled corners to simplify the calculations.

$$\mathrm{J} = \frac{a^4}{6} - \frac{b^4}{6} = \frac{a^4 - b^4}{6} \tag{2.1,r}$$

$$\mathrm{J} = \frac{a^4}{6} - \frac{b^4}{6} = \frac{a^4 - b^4}{6} = \frac{0.05^4 - 0.044^4}{6} \cong 4.17 \cdot 10^{-7}\ m^4 \tag{2.2,r}$$

The shear modulus from the axle is taken from Granta EduPack, and has a range of 78.9 – 82.7 GPa.

The angle of twist is represented in degrees for easier interpretation, (2.3) gives the conversion from radians to degrees.

$$\theta = \frac{360}{2 \cdot \pi} \cdot \phi \tag{2.3}$$

Thus the angle of twist as a function of single applied torque is determined via (2.4), where the angle is expressed in degrees. The distance taken as the furthest distance as this results in the largest angle of twist, this distance is longer as the distance used in the previous calculations, this is due to the axle being extended into the gearbox. The shear modulus is taken as 78.9 GPa, as this again results in the largest angle of twist.

$$\theta(\mathrm{T}) = \frac{T \cdot L}{G \cdot J} \cdot \frac{360}{2 \cdot \pi} = \frac{T \cdot 0.6}{78.9 \cdot 10^9 \cdot 4.17 \cdot 10^{-7}} \cdot \frac{360}{2 \cdot \pi} \cong 0.001T \tag{2.4}$$

For the current machine the maximum torque the motor can deliver to the axle is determined via (2.5), as the rotation speed is known as 700 rpm (~73.3 rad/s) with a power rating of 0.55 kW. The axle rotates at 26 rpm.

$$T_{axle} = \frac{P}{\omega} \cdot i = \frac{550}{73.3} \cdot \frac{700}{26} \cong 202\ \mathrm{Nm} \tag{2.5,r}$$

The maximum torque expected to be present, if the motor were able to power, would be 4500 Nm. This results in a torsion angle of 4.5°, see (2.4). This is smaller than the angles between the blades, and therefore adequate, as only one blades thus comes in contact for the shredding. Please note, the axle fails over ~ 1600 Nm according (2.6).

2.2 Angular Stress

The maximum shear stress in the beam due to the applied torque is determined via (2.6). The shear strength of the axle is taken from Granta EduPack and has a range of 136 – 158 MPa.

$$\tau_{max} = \frac{T \cdot y}{J} = \frac{202 \cdot 0.0353}{4.17 \cdot 10^{-7}} \cong 17.1 \text{ MPa} \tag{2.6r}$$

The maximum shear stress is substantially smaller than the shear strength, thus the axle will not fail due to shear stress produced by the applied torque. By rewriting (2.6) it is calculated that this axle will fail due to shear stress coming from the torque at an applied torque of ~1600 Nm, where the shear strength is taken as 136 MPa.