Predicting the Behavior of a Rat Trap Based on a Simplified Physical Model

Physical Model

The following is an analysis of a rat trap from the point it is released until the bar slams shut against the wooden base. The analysis uses a simplified model based on the physics of ideal springs and simple harmonic motion.

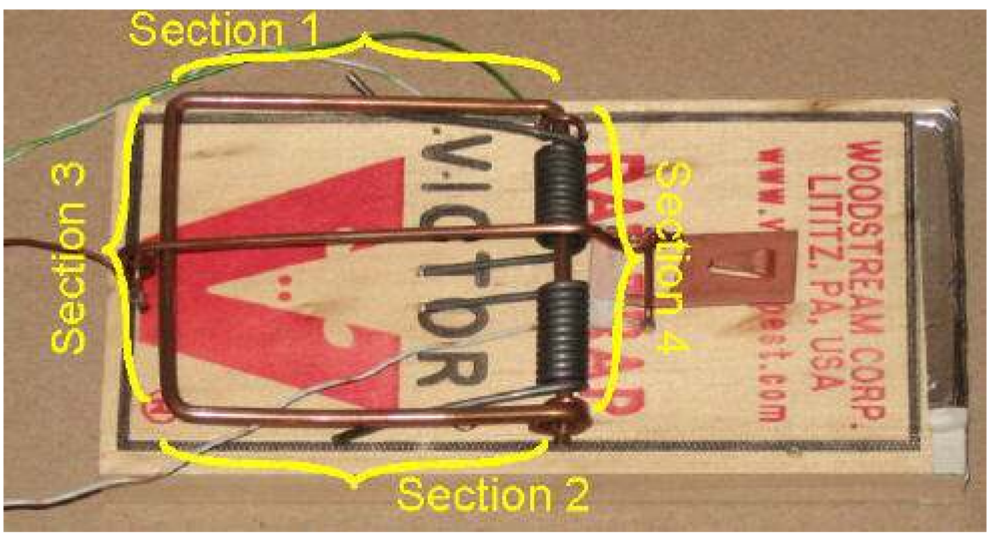
In this simplified analysis, many things have been idealized. Frictional forces, air resistance, and gravitational forces have been neglected. The energy used to push the hook up and out of the way has not been considered. The spring is assumed to be ideal, where the torsion constant does not change with the angle.

Moment of Inertia of the Bar

One of the most important parameters that you need to determine is the moment of inertia of the bar. This parameter is used in the analysis of many situations where rotational motion is being examined. It is a characteristic of a rotating object, and it depends on the geometry and mass of the object and the axis about which it is rotated. Moment of inertia is analogous to the mass of an object when dealing with linear motion.

The bar consists of a rectangular wire loop, and the moment of inertia contribution of each of the four sides of the loop must be determined separately. For simplicity the wire of the loop will be considered to be "infinitely thin".

The picture below identifies each of the four sides.



Contribution from sections 1 and 2

The contributions of sections 1 and 2 are identical, as they are for all practical purposes identical in geometry and mass and they rotate about the same point.

The moment of inertia of a single thin rod rotating about one end is:

$$I = \frac{1}{3} \cdot M \cdot L^2$$

Where M is the mass of the rod segment and L is the length of the rod. This equation is used to determine the contribution from each of the two sides of the rectangular wire loop that forms the bar.

Contribution from section 3

The moment of inertia contribution from the front portion of the loop is determined differently. It is equal to the mass of that section, multiplied by the square of the length L.

$$I = M \cdot L^2$$

Where M is the mass and L is the length of rod segment 3.

Contribution from section 4

The moment of inertia contribution from this part of the loop can be neglected, as its contribution is minimal compared to that of the other three sections of the loop. Its mass is concentrated much closer to the axis of rotation than that of the other segments, so its contribution is much smaller.

Torsion Constant of the spring

The torsion constant of the spring is a value that determines how much torque will be required to move the spring through a given angle. It is expressed in torque per angle. In the case of an ideal spring, it is a constant and does not change with the position angle.

This constant needs to be calculated from measurements. For the trap, it was determined by pulling the bar with a spring scale while keeping the angle between the applied force and the bar equal to 90 degrees. The torque value is equal to the force measured multiplied by the length of the bar.

If two torque measurements are made, the torsion constant is determined by dividing the difference in torque by the difference in angle:

$$k = \frac{\tau_2 - \tau_1}{\theta_2 - \theta_1}$$

Equations of Motion

The spring exerts torque on the bar. The torque is equal to the moment of inertia multiplied by the angular acceleration.

$$\tau = I \cdot \alpha$$

The angular acceleration is the second derivative of θ with respect to time, so this can also be written as:

$$\tau = I \cdot \frac{d^2\theta}{dt^2}$$

The torque is determined by the product of the torsion constant and the angle, so:

$$-k\theta = I \cdot \frac{d^2\theta}{dt^2}$$

Rearranging we get:

$$\frac{d^2\theta}{dt^2} = \frac{-k\theta}{I}$$

The solution to the equation above is a sinusoidal function:

$$\theta(t) = \Theta \cdot \cos(\omega \cdot t)$$

This is the equation for the angular position of the bar as a function of time. In the equation, Θ is the full scale angular displacement of the motion, and ω is the frequency of the oscillation.

In the case of the trap, the full scale angular displacement is the angle between the wooden base and the bar when the trap is set. It is almost 180 degrees for a rat trap.

The frequency of oscillation is determined by the torsion constant k of the spring and the moment of inertia I of the bar as follows:

$$\omega = \sqrt{\frac{k}{I}}$$

This is a case of simple harmonic motion. If the bar could move unobstructed, it would oscillate back and forth in simple harmonic motion. In the ideal case considered here, where there is no friction, etc. the oscillation would continue forever.

In the case of the trap, the bar is abruptly stopped in the middle of the first half cycle of oscillation when it slams into the wooden base. Even through the bar will not oscillate more than ¼ of a cycle, the equations of simple harmonic motion can be used to show its how its position and velocity change over time.

Potential Energy

It is also useful to examine the potential energy stored in the spring. When the trap is set, potential energy is stored in the spring. In the idealized case, all the stored potential energy will be converted into kinetic energy as the bar is released and it accelerates toward the wooden base.

The change in potential energy stored in the spring as a function of the angle through which it is moved is determined by:

$$P_E = \int_{\phi_1}^{\phi_2} \tau d\theta$$

If the torque is linearly related to the angle, which is the case of the ideal spring, then the integral results in the following formula:

$$P_E = \frac{1}{2} \cdot k \cdot (\theta_2^2 - \theta_1^2)$$

This represents the energy stored in the spring when moving from angle θ_1 to θ_2 .

Kinetic Energy

The kinetic energy of a rotating object is given by:

$$K_E = \frac{1}{2} \cdot I \cdot \omega^2$$

Where ω is the angular velocity.

A maximum value for the angular velocity of the bar can be obtained using conservation of energy. If the kinetic energy is set equal to the potential energy stored in the spring and you solve for the angular velocity, the equation is:

$$\omega = \sqrt{\frac{2 \cdot P_E}{I}}$$

This is the maximum angular velocity in radians per second, if all of the stored potential energy is converted into kinetic energy. In the real world there will be losses of energy that will make the maximum velocity somewhat lower.

Results:

The next page contains a spreadsheet of the results of the analysis of a rat trap, based on the equations presented above.

When the time to close value was measured on a real trip, the observed value was 23 milliseconds, whereas the value predicted using the model presented here is 14.5 milliseconds. In an actual trap, frictional losses would reduce the speed, so we would expect the value calculated for the ideal case to be faster.

Resulting Calculations for a Victor Rat Trap.

Bar Dimensions and other Parameters					
A=	2.79	in	0.0709	m	Width of wire loop (bar)
B=	3.203	in	0.0814	m	Length of wire loop (bar)
wire_dia=	0.135	in	0.0034	m	Wire diameter of wire loop (bar)
density=	8	g/cm3	8000	kg/m3	Assumed density of wire material

Mass of bar sections			
M1=	0.00601	kg	Mass of bar section 1
M2=	0.00601	kg	Mass of bar section 2
M3=	0.00524	kg	Mass of bar section 3

Moment of inertia of bar sections			
I1=	1.326E-05	kg*m2	Moment of inertia of bar section 1
I2=	1.326E-05	kg*m2	Moment of inertia of bar section 2
I3=	3.465E-05	kg*m2	Moment of inertia of bar section 3
I_total=	6.117E-05	kg*m2	Total moment of intertia

Force and Torque					
F1=	6.5	lbs	28.364	N	Force on front of bar, when set
F2=	0.5	lbs	2.182	N	Force on front of bar, when closed
T1=	2.308	N*m			Torque on bar when set
T2=	0.178	N*m			Torque on bar when closed
Th_1	170	deg	2.967	rad	Angle of bar when set
Th_2	0	deg	0	rad	Angle of bar when closed
Th_0=	-3.247	deg	-0.057	rad	Angle at which the spring is completely relaxed
	173.247	deg	3.024	rad	Full scale angular displacement of bar

Torsion Constant			
k=	0.71790	N*m/rad	Spring torsion constant

Potential Energy			
E=	3.16	Joules	Potential energy stored in spring when set

Angular Frequency					
w=	108.330	rad/sec	17.241	rev/sec	Frequency of oscillation (simple harmonic motion)
			58.000	millisec/rev	Time of a single cycle (simple harmonic motion)
			14.500	milliseconds	Time to close (bar impacts wood base)

Final Angular Velocity					
	321.422	rad/s	51.156	rev/sec	Final rotational velocity of bar before impact. (Determined by setting potential energy equal to final kenetic energy and solving for the angular velocity.

Final Linear Velocity					
	85.793	ft/sec	58.495	miles/hour	Linear velocity of front of bar, prior to impact with the base

The following graphs show the calculated results for the angular position of the bar with respect to time, the angular velocity with respect to time, and the linear speed of the front of the bar (section 3) with respect to time. The time scale is in milliseconds. The red line shows the point at which the bar strikes the wooden base.

The portions of the graphs beyond that point represent how the system would behave if it could swing back and forth in simple harmonic motion, unimpeded.

Note that the peak linear velocity of the bar is just over 50 miles per hour prior to impact!