Power Calculations

This document describes the power calculations for the shredder. The third chapter shows how to quickly calculate a shredder setup via a script, within MATLAB:

1. Required power for given material.
2. Shredding ability of machine with given power.
3. MATLAB scripts.

The calculations use the shear strength of plastics to determine the power. Plastic are not isotropic and vary wildly in their shear strength by consequence of various parameters. For the calculations the shear strength is taken to be equal to tensile strength. Table 1 gives the tensile strength, and thus assumed shear strength of various plastics:

Code	Description	Tensile Strength	Unit
PMMA	Polymethyl Methacrylate (Acrylic)	54 - 72	MPa
PLA	Polylactide	55 - 72	MPa
PP	Polypropylene	26 – 50	MPa
PS	Polystyrene	35.9 – 51.7	MPa
PET	Polyethylene Terephthalate	55 - 60	MPa
PE	Polyethylene	20.7 – 44.8	MPa
ABS	Acrylonitrile Butadiene Styrene	37.9 – 51.7	MPa

Table 1 *Tensile strength of various plastics (values from Granta EduPack)*

1. Required Power

The required power is determined via the following described method. This method does not take the possible deformation of the plastic into consideration. The power is determined via (1.1).

$$P = T \cdot \omega \tag{1.1}$$

Where P represents the power in Watts, T the torque in Nm and ω the angular velocity in rad/s. The power is determined by taking the angular velocity of the axle of the shredder times the torque on the axle of the shredder. The power can also be determined by taking the torque on the motor axle times the angular velocity of the motor axle. They are **not** interchangeable.

$$P_{motor-axle} = T_{motor-axle} \cdot \omega_{motor-axle} \tag{1.2a}$$

$$P_{motor-axle} = T_{shredder-axle} \cdot \omega_{shredder-axle} \tag{1.2b}$$

Thus, the power in the shredder axle and the motor axle is the same. The latter only applies if the transmission has no energy losses. The torque on the shredder axle can thus be increased by decreasing the angular velocity of the shredder axle with a transmission.

$$\omega_{shredder} = \frac{\omega_{motor}}{i} \tag{1.3}$$

Where i represents the gearing reduction of the transmission. The rotational speed of the motor is generally given in rounds per minute. The conversion from RPM to rad/s is given in (1.4). Where n is the rotational speed in rounds per minute.

$$\omega = n \cdot \frac{2 \cdot \pi}{60} \tag{1.4}$$

The torque required for the shredding of the plastics is determined via (1.5). Where F is the force in Newton and r the radius from center of the axle to the point where the force is applied in meters.

$$T = F \cdot r \tag{1.5}$$

$$F = \sigma_{ss} \cdot A = \left[\frac{N}{m^2}\right] \cdot [m^2] = [N] \tag{1.6}$$

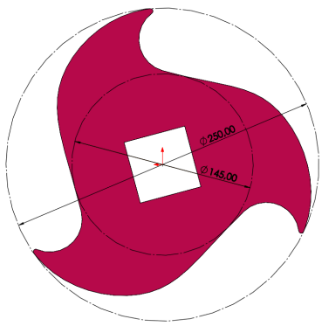
The force is acquired via (1.6), by taking the shear strength of the material and multiplying it by the area to be sheared. The brackets show the units of the variables for ease of understanding.

The torque is thus proportional to the sheared surface. The area is determined by taking the circumference of the first contact of the blade and multiplying this by the thickness of the material. This initial contact is expected to result in the highest torque during the shredding process, as the largest circumference is at the start.

$$c_{blade} = 2 \cdot (w + h) \tag{1.7}$$

Where c is the circumference of the shearing blade in meters, w the width of the blade and h the height of the shearing surface contact, both in meters.

Table 2 gives the values for the blade in the shredder. The figure shows some of the dimensions used during the power calculations for ease of understanding.



Code	Description	Value	Unit
$\boldsymbol{w}$	Width of the blade	0.03	m
$\boldsymbol{h}$	Height of the blade at shearing circumference	0.004	m
$\boldsymbol{r}$	Radius from center of blade to applied force	0.125	m

Table 2 *Characteristics of the shearing surface of blade*

Thus, the required power for the shredding of a material is seen in equation (1.8). Where z represents the number of blades that are shredding simultaneously and t the thickness of material.

$$P = F \cdot r \cdot \omega = (\sigma_{ss} \cdot c_{blade} \cdot z \cdot t) \cdot r \cdot \omega_{shredder-axle} \tag{1.8}$$

With this the power for the shredding of plastics is calculated. For the design the shredding of 6 mm PMMA plates would result in (1.8,r). The speed of the shredder axle is 10 rpm.

$$P = 72 \cdot 10^6 \cdot 2 \cdot (0.03 + 0.004) \cdot 1 \cdot 0.006 \cdot 0.125 \cdot 10 \cdot \frac{2 \cdot \pi}{60} \cong 3.8\, kW \tag{1.8,r}$$

2. Shredding Ability

The ability of a shredder to shred a certain thickness of a known material with a given input power is determined via (2.1). This equation is a rewritten form is the complete calculation discussed in chapter 1.

$$t = \frac{P}{(\sigma_{ss} \cdot c_{blade} \cdot z) \cdot r \cdot \omega_{shredder-axle}} \tag{2.1}$$

The existing shredder has an shredder axle speed of 27 rpm and a power output of 550 W, this results in a maximum thickness of PMMA of 0.3 mm (2.1,r), which is inadequate.

$$t = \frac{550}{72 \cdot 10^7 \cdot 2 \cdot (0.03 + 0.004) \cdot 2 \cdot 0.125 \cdot 27 \cdot \frac{2 \cdot \pi}{60}} \cong 0.3\, mm \tag{2.1,r}$$

3. MATLAB Scripts

Every small change of a parameter in the shredder gives vastly new results in performance. As there are many steps in the calculations, it is advised to use a script to quickly calculate the results. The script can also be used to compare different shredding setups and find the best solution. Below the scripts for both chapters 1 and 2 are found.

```
clc;
clear;
close;

% Materials Tensile Strength (Maximum value Granta EduPack)
PMMA = 72 * 10^6;    % PMMA
PLA = 72 * 10^6;     % PLA
PP = 50 * 10^6;      % PP
PS = 51.7 * 10^6;    % PS
PET = 60 * 10^6;     % PET
PE = 44.8 * 10^6;    % PE
ABS = 51.7 * 10^6;   % ABS

% Material Properties & Selection
Tensile_Strength = PMMA;                 % Pa [N / m^2]
x = 1;                                   % Shear Strength derived from Tensile Strength [0-1]
Shear_Strength = Tensile_Strength * x;   % Pa [N / m^2]
t = 0.006;                               % Thickness of plate material

% Calculating the shear area per blade
width = 0.03;                           % Width of blade [m]
height = 0.004;                          % Height of blade at shear circumference [m]
r = 0.125;                               % Radius to shear force [m]
c = 2*(width + height);                  % Circumference of shearing blade [m]
A = c * t;                               % Shear area [m^2]

% Required force & torque per blade
F_blade = Shear_Strength * A;            % Shear force on blade [N]
T_blade = F_blade * r;                   % Shear torque on blade [Nm]

% Required power
z = 1;                                           % Blades that shear simultaneously
T_total = T_blade * z;                           % Total torque from shearing blades [Nm]
rpm_shredder = 10;                               % Angular velocity shredder [rpm]
w_shredder = rpm_shredder * ((2*pi) / 60);       % Angular velocity shredder [rad/s]
P_required = T_total * w_shredder;               % Required power [W]
disp(P_required);                                % Displays the value of required power
```

Figure 1 *Script for calculating required power as discussed in chapter 1*

```
clc;
clear;
close;

% Materials Tensile Strength (Maximum value Granta EduPack)
PMMA = 72 * 10^6;    % PMMA
PLA = 72 * 10^6;     % PLA
PP = 50 * 10^6;      % PP
PS = 51.7 * 10^6;    % PS
PET = 60 * 10^6;     % PET
PE = 44.8 * 10^6;    % PE
ABS = 51.7 * 10^6;   % ABS

% Material Properties & Selection of parts
Tensile_Strength = PMMA;                % Pa [N / m^2]
x = 1;                                  % Shear Strength derived from Tensile Strength [0-1]
Shear_Strength = Tensile_Strength * x;  % Pa [N / m^2]
P_motor = 550;                          % Power of the motor [W]
z = 1;                                  % Number of blades shearing simultaneously
rpm_shredder = 27;                      % Angular velocity of axle shredder [rpm]

% Calculating the shear area per blade
width = 0.03;                           % Width of blade [m]
height = 0.004;                         % Height of blade at shear circumference [m]
r = 0.125;                              % Radius to shear force [m]
c = 2*(width + height);                 % Circumference of shearing blade [m]

% Possible area to be sheared
w_shredder = rpm_shredder * ((2*pi)/60);    % Angular velocity axle shredder [rad/s]
T_shredder = P_motor / w_shredder;          % Total torque in available on shredder axle [m]
T_blade = T_shredder / z;                   % Torque available per blade [Nm]
F_blade = T_blade / r;                      % Force per blade [N]
A_available = F_blade / Shear_Strength;     % Available area to be sheared with given power [m^2]
t = A_available / c;                        % Maximum thickness possible to shred [m]
t_mm = t * 1000;                            % Maximum thickness possible to shred [mm]

disp(t_mm);
```

Figure 2 *Script for calculating maximum material thickness as discussed in chapter 2*