When Water Falls:
Rayleigh-Plateau Instability

Overview

Although the name may sound intimidating, the Rayleigh-Plateau instability phenomenon occurs in the most ordinary of places. Whether it be when the rain falls from the sky or when you turn a faucet on to fill a cup of water, this phenomena can be witnessed from anywhere. By analyzing this phenomena we can determine when, at some critical length, the fluid loses its cylindrical, “smooth” shape and breaks apart into individual droplets (just like rain falling from the sky). This happens because any small displacement at the origin of the stream amplifies along the length of the stream, and forming droplets due to these displacements is a way to minimize surface area and energy of the fluid flow.

Scientists and engineers have been studying this phenomenon for years, finding applications in optical fiber manufacturing, printing, and even black holes. But today we will take a step back from these high level applications and take a look at the simplest form of the phenomena that we have been experiencing our entire lives without even realizing it. By the end of this experiment you (as a budding scientist) will understand the forces and properties that affect this fluid phenomena. This experiment will highlight Rayleigh-Plateau instability and demonstrate some of the factors that influence the point at which the stream breaks apart.

Objectives

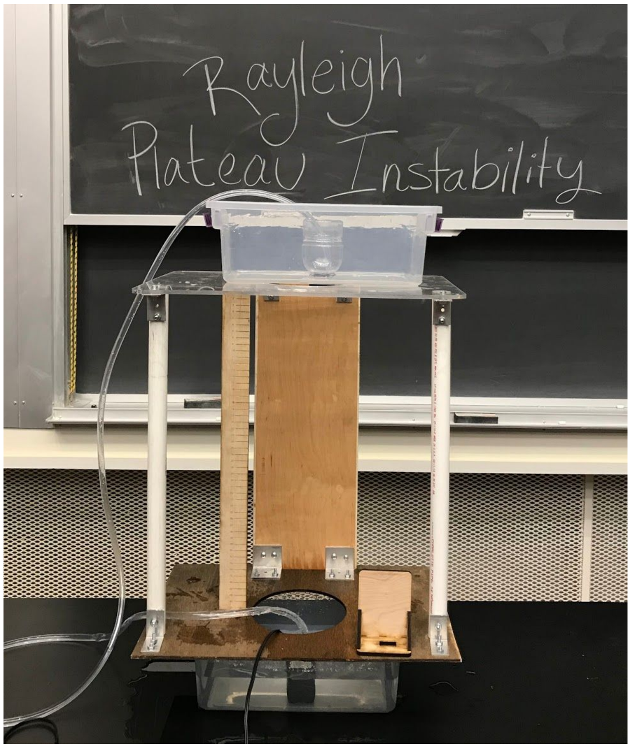
- Identify, using a measurement device, the location where the stream forms water droplets, known as the critical length
- Observe the effect of a changing nozzle radius on the critical length of water
- Attempt to match the frequency of water droplet formation with the frequency of a strobe light to “freeze” droplets in space

Materials

- Rayleigh-Plateau instability demonstration apparatus
    - 2 nozzles with radii .25 cm and .5 cm
- 250 ml isopropyl alcohol
- deionized (DI) water (tap water is acceptable but results are less likely to match theory)
- 1 smartphone with free strobe light app (App store/Google Store have plenty of options)
- 1 smartphone with slow motion video capabilities

Experimental Setup

Please see the written instructables for a walkthrough of the design of the setup.



Procedure

1. Place a nozzle in the opening in the upper fluid tank and set the end of the tubing in the nozzle
2. Clean the system (i.e. tanks, tubes, nozzles) with the isopropyl alcohol. This cleaning will allow for the most accurate results. Dispose of the isopropyl alcohol as instructed by your teacher.
3. Secure the pump to on the bottom of the lower fluid tank. Fill the lower fluid tank with deionized water so that the pump is completely submerged and covered by an additional inch or so of water.
4. Screw the 0.5 cm radius cap on your nozzle, making sure it creates a tight seal.
5. Turn on the pump and adjust pump rate so that the nozzle fills with DI water. You will start to see the fluid stream and the instability form.
6. Once the nozzle is full, hold a graduated cylinder under the stream and collect water for 5 seconds. Record the volume of water collected in that time.
7. Start the strobe app and point the light at the fluid stream. Take a slow motion video of the stream for at least 5 seconds. Do this three times, ensuring that the water level in the nozzle stays constant.
8. Turn off the pump and let all the water fall back into the lower fluid tank.
9. Repeat steps 6-9 for the .25 cm nozzle.
10. Now, with the strobe light app, see if you can illuminate the water droplets with a strobing frequency that matches the frequency of droplet formation. This step will require a lot of trial and error so be patient. If done correctly, it will look like you have frozen a water droplet in time and space!

    Tip: for the 0.5 cm nozzle, try flashing frequencies between 200 and 500 Hz.

11. Once all measurements and necessary photographs have been taken, empty the fluid tanks into the sink and clean up materials according to your teacher's instructions.

Analysis

1. Using the smartphone footage, identify 3 clear frames for each trial and record the critical length.
2. Report the average critical length for each nozzle from all trials.
3. Find the speed of the fluid stream (U) in m/s. Using your data from step 6, calculate the volume of water that leaves the nozzle per second in mL/s: this is the volumetric flow rate (Q). Then, calculate the exit area (A) of the nozzle based on the hole radii. From there, use the below equation to find the speed of fluid leaving the nozzle.

$$U = Q/A$$

4. At 20°C, deionized water has a density (ρ) of approximately 1.0 g/mL and a surface tension (γ) of approximately 0.72 N/m. Researchers have found that radius and critical length are related by this equation:

$$L_{crit} = c * U * \sqrt{\frac{\rho R^3}{\gamma}}$$

where c is some unknown constant. Using your data, solve for c for each nozzle.

5. Let's do some math to see how changing the radius should affect the critical length. Let's consider the equation $L_0 = c * U * \sqrt{\frac{\rho {R_0}^3}{\gamma}}$, where L_0 and R_0 are known variables. If we change the nozzle such that the new radius, R', is twice R_0, find the new critical length L' and compare it to the original. *(Hint: plug in $R' = 2 * R_0$ and then find L' in terms of $L_0, c, U, \rho, \gamma, and\ R_0$)*

6. Compare this factor to the experimental results you collected and calculate the percent error.
7. Finally, analyze your recorded results. If you wanted the stream to stay smooth for as long as possible, which nozzle radius would you want to use? Does this mathematical reasoning match what you observed in the experiment?

   What if we used a different substance other than water. Would a denser liquid be better or worse for making the critical length longer? (*hint: look at the equation above relating critical length to the four different parameters*)