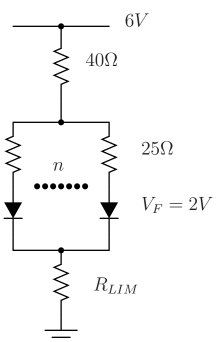
All references in this writeup are to the Atmel ATtiny2313/V Preliminary Reference Manual.

1 Choosing the LED current limiting resistor

At most two LEDs are on at any time and, by design, the duty cycle is at most 20%. There are two main constraints; (1) the maximum DC pin current must be $\leq$ 40mA, and (2) the total high pin current must be $\leq$ 60mA. (It's not entirely clear if the latter constraint is a DC or transient constraint, we will treat it as transient, which is more stringent.)

The CR2032 batteries have a typical internal resistance of 20Ω, hence the supply resistance is 40Ω. Fig. 105 suggests that the driving pin resistance is about 25Ω.

The total LED current required to drive n LEDs simultaneously can be estimated using the following equivalent circuit:



The transient constraint gives:

$$\frac{6-2}{40+\frac{25}{n}+R_{LIM}} \leq 60 \text{ mA} \tag{1}$$

or, equivalently:

$$R_{LIM} \geq 26.\dot{6} - \frac{25}{n}\Omega \tag{2}$$

Letting $n = 2$ gives the worst case (by design), which is $R_{LIM} \geq 14.1\dot{6}\Omega$. The nearest value I have in my box is $R_{LIM} = 10\Omega$, which should be fine. The estimate above gives a total LED current of 53mA if one LED was lit, and 64mA if two LEDs were lit. The DC constraint is easily satisfied since the maximum DC current *per pin* is $\leq (0.2)64 = 12.8$mA (since the duty cycle is at most 20%).

The 64mA is slightly higher than the total pin current constraint, which is fine in this context, but just for giggles, a better estimate can be made by using a load-line analysis. (A 60mA supply current will result in a significant drop because the CR2032 internal resistance is fairly high.) Fig. 105 on p.196 gives the driving point I-V characteristic for a pin driven high. This characteristic can be nicely modeled by a quadratic, and the LED modelled as a drop of V_F. Superimposing the lines in Fig. 1 shows that the above estimate was generous, the total LED current based on a load-line analysis is 40mA if one LED was lit, and 48mA (twice the single LED current) if two LEDs were lit.

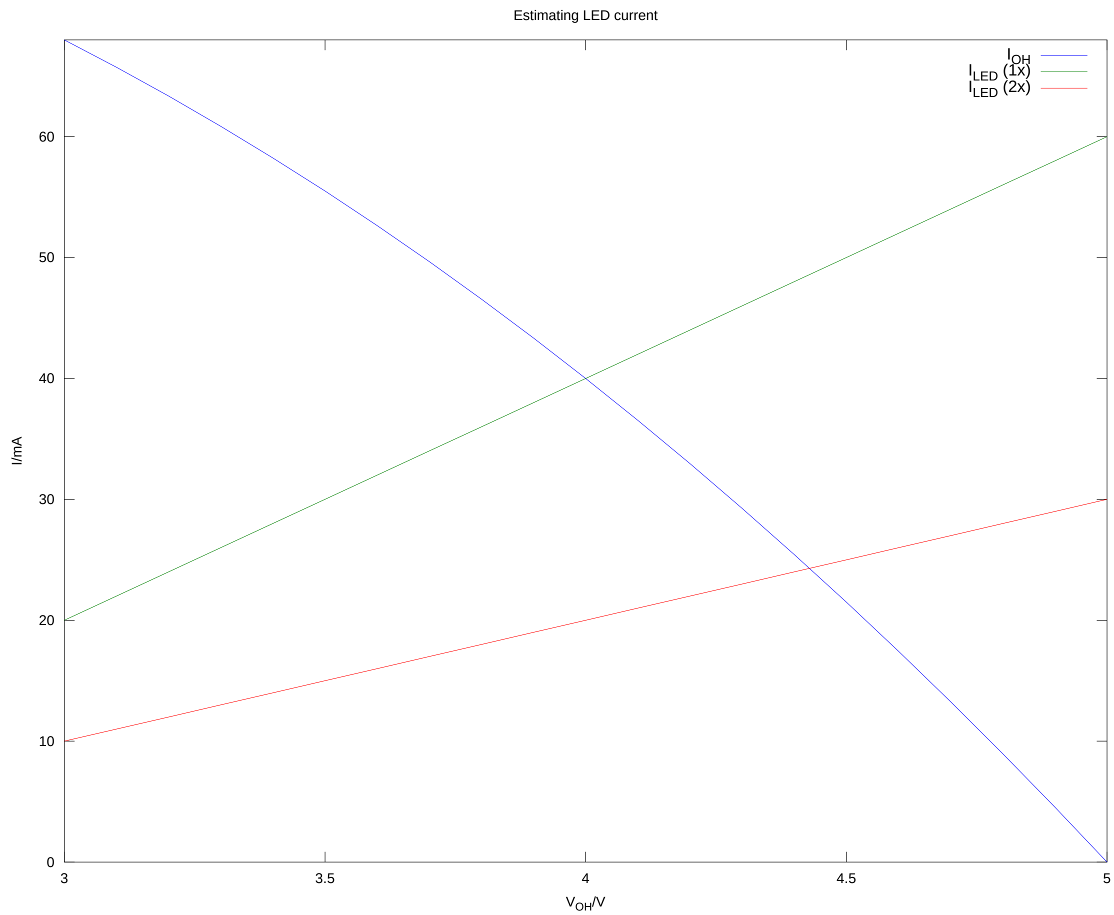
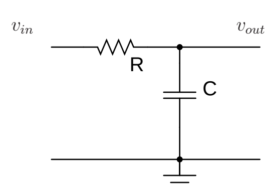


Figure 1: LED current estimation.

I should be careful to point out that the above load-line analysis is still very rough, as it assumes that the driving point characteristic of the pin is unaffected by the supply voltage (which is affected by the current draw). A quick look at the specs. shows that this is not true, however the driving point characteristics for lower supplies result in a lower current, so this is still a slightly conservative estimate.

In fact, a load-line analysis shows that removing R_{LIM} entirely would also remain under the 60mA constraint. However, I figured this out after I had built the circuit!

2 Analysing the RC charge/discharge circuit



For a reasonable input voltage $v_{in}(.)$, the output voltage $v_{out}(.)$ of the RC circuit is given by the formula:

$$v_{out}(t) = v_{out}(0)e^{-\frac{t}{RC}} + \frac{1}{RC}\int_0^t e^{-\frac{t-\tau}{RC}} v_{in}(\tau)d\tau \tag{3}$$

If the input voltage is a constant V_{in}, then this can be simplified to:

$$v_{out}(t) = v_{out}(0)e^{-\frac{t}{RC}} + V_{in}(1 - e^{-\frac{t}{RC}}) \tag{4}$$

There are two cases of interest to us; charging and discharging. In the case of charging, $v_{out}(0) = 0$ (assuming the low output, $V_{OL} = 0$), and $V_{in} = V_{OH}$, which gives:

$$v_{out}(t) = V_{OH}(1 - e^{-\frac{t}{RC}}) \tag{5}$$

In the discharging case, we take $v_{out}(0) = V_{OH}$, and $V_{in} = 0$, giving:

$$v_{out}(t) = V_{OH}e^{-\frac{t}{RC}}. \tag{6}$$

In the above, I use V_{OH} rather than V_{CC} as the high voltage, mainly as a reminder that the coin cell IR drop is fairly large, so V_{OH} and V_{CC} can differ by more than usual.

3 Choosing R_{DIS}

The capacitor should be discharged as quickly as possible, the limitations are that the maximum DC pin current must be $\leq$ 40mA, and the maximum transient current must be $\leq$ 60mA. During discharge the pin current is given by

$$i_{pin}(t) \quad = \quad \frac{V_{OH}}{R}e^{-\frac{t}{RC}}, \tag{7}$$

so the maximum current is $\frac{V_{OH}}{R}$, and the average discharge current can be computed by the following (in fact, this is an overestimate, since the discharges are not back to back), where k is the number of time constants over which the discharging occurs

$$\begin{aligned} \bar{i}_{pin} &\leq \frac{1}{kRC}\int_0^{kRC} i_{pin}(\tau)d\tau & (8) \\ &= \frac{1}{kRC}\frac{V_{OH}}{R}\int_0^{kRC} e^{-\frac{\tau}{RC}}d\tau & (9) \\ &= \frac{V_{OH}}{R}\frac{(1-e^{-k})}{k} & (10) \end{aligned}$$

This shows that for $k \geq 2$, $\bar{i}_{pin} \leq \frac{1}{2}\frac{V_{OH}}{R}$, and so the transient constraint is harder to satisfy than the DC constraint. To satisfy the transient constraint, we need

$$\frac{V_{OH}}{R} \leq 60\text{mA} \tag{11}$$

or, in terms of the resistance R,

$$R \geq \frac{V_{OH}}{60\text{mA}} \tag{12}$$

Since $V_{OH} \leq 6V$, this results in $R \geq 100\Omega$.

4 Relationship of time to measured voltage

Formula 5 can be rewritten as

$$\frac{v_{out}(t)}{V_{OH}} = (1 - e^{-\frac{t}{RC}}) \tag{13}$$

The conversion time t_c is the time taken for $v_{out}(t)$ to match the divided load resistor voltage v, hence

$$\frac{v}{V_{OH}} = (1 - e^{-\frac{t_c}{RC}}) \tag{14}$$

from which we get

$$t_c = RC\ln(\frac{1}{1 - \frac{v}{V_{OH}}}) \tag{15}$$

This can be used to (loosely) estimate the sensitivity of t_c to variations in V_{CC}. The divided load resistor voltage v is given by

$$v = \frac{22}{32}(V_{CC} - R_L i_d) \tag{16}$$

which gives

$$\frac{v}{V_{CC}} = \frac{22}{32}(1 - \frac{R_L i_d}{V_{CC}}) \tag{17}$$

So, abusing notation a little (and noting that in the saturation region i_d is roughly independent of V_{CC}) we have

$$\frac{d(\frac{v}{V_{CC}})}{dV_{CC}} = \frac{22}{32}\frac{R_L i_d}{{V_{CC}}^2} \tag{18}$$

Plugging in $V_{CC} = 5V$, $i_d = 9\text{mA}$ and $R_L = 330\Omega$ shows that the sensitivity is less than $0.03V^{-1}$.

Assuming that changes in V_{OH} are the same as changes in V_{CC}, we can use Formula 15 to estimate the effect of changes in V_{CC} on t_c. Abusing notation again, we have

$$\frac{dt_c}{d(\frac{v}{V_{OH}})} = RC\frac{1}{1 - \frac{v}{V_{OH}}} \tag{19}$$

By design, we have $v \leq \frac{22}{32}V_{OH}$, so we have

$$\frac{dt_c}{d(\frac{v}{V_{OH}})} \leq 3.2RC \tag{20}$$

Plugging in $RC = 470\mu\text{S}$ gives $\frac{dt_c}{d(\frac{v}{V_{OH}})} \leq 1504$, from which we can estimate that the impact of the supply voltage change on the conversion time is about $45\mu\text{S/V}$, which is significant, but not a huge issue for this application.